SR02 Mass-Energy & Einstein

SPH4U

CH 11 (KEY IDEAS)

- State Einstein's two postulates for the special theory of relativity.
- Describe Einstein's thought experiments demonstrating relativity of simultaneity, time dilation, and length contraction.
- State the laws of conservation of mass and energy, using Einstein's mass–energy equivalence.
- Conduct thought experiments involving objects travelling at different speeds, including those approaching the speed of light.

EQUATIONS

- Total Energy

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Rest Energy

$$E_{\text{rest}} = mc^2$$

CHANGES IN KINETIC ENERGY

- Work done on an object by definition changes its kinetic energy
  - Recall: work is product of the parallel component of force and displacement ($W = Fd\cos\theta$)
- This can be represented as

$$W = \Delta E_K = m\left(v_f^2 - v_i^2\right)$$

- However, if mass remains constant, all the energy provided by the force goes into increasing speed
- This relationship breaks down once we get to relativistic speeds (speeds approaching the speed of light)

TOTAL ENERGY AND REST ENERGY

- Einstein suggested that the total energy of an object of rest mass m travelling at speed v relative to an inertial frame is

$$\boldsymbol{E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

- When the object is at rest in the inertial frame, $v = 0$ and the energy is

$$\boldsymbol{E_{rest} = mc^2}$$

RELATIVISTIC KINETIC ENERGY

- **Conservation of Mass–Energy:** the principle that rest mass and energy are equivalent
- Relativistic Kinetic Energy: the extra energy given to an object as a result of its motion

$$E_{\text{total}} = E_{\text{rest}} + E_K$$

$$E_K = E_{\text{total}} - E_{\text{rest}}$$

$$E_K = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

CHANGING MASS INTO ENERGY

- For an object at rest, we have Einstein's relationship

$$E_{rest} = mc^2$$

- If there is a change in energy there must be a corresponding change in mass, since c is constant, or

$$\Delta E = \Delta mc^2$$

$$\Delta m = \frac{\Delta E}{c^2}$$

- This relationship can help us predict the available energy available in a quantity of matter

PROBLEM 1

If the 0.50 kg mass of a ball at rest were totally converted to another form of energy, what would the energy output be?

PROBLEM 1 – SOLUTIONS

$\Delta m = 0.50 \text{ kg}$

$\Delta E = ?$

$$\Delta E = (\Delta m)c^2$$
$$= (0.50 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$
$$\Delta E = 4.5 \times 10^{16} \text{ J}$$

The energy equivalent is 4.5×10^{16} J, or approximately half the energy emitted by the Sun every second.

PROBLEM 2

In subatomic physics, it is convenient to use the electron volt, rather than the joule, as the unit of energy. An electron moves at $0.860c$ in a laboratory. Calculate the electron's rest energy, total energy, and kinetic energy in the laboratory frame, in electron volts.

PROBLEM 2 – SOLUTIONS

$m = 9.11 \times 10^{-31}$ kg

$v = 0.860c$

$E_{rest} = \ ?$

$E_{total} = \ ?$

$E_K = \ ?$

$$E_{rest} = mc^2$$

$$= (9.11 \times 10^{-31}\text{ kg})(3.00 \times 10^8\text{ m/s})^2$$

$$E_{rest} = 8.199 \times 10^{-14}\text{ J}$$

Converting to electron volts:

1.60×10^{-19} J = 1 eV

$$(8.199 \times 10^{-14}\text{ J})\,\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ J}} = 5.12 \times 10^5\text{ eV}$$

PROBLEM 2 – SOLUTIONS CONT.

This result is still a large number, so it is convenient to express the energy in mega electron volts (MeV), giving the result 0.512 MeV.

Reference books, however, show that the accepted rest energy of an electron is 0.511 MeV. Where did we go wrong? Our calculation assumed a value of 3.00×10^8 m/s for c. As noted earlier, a more precise value for c is $2.997\,924\,58 \times 10^8$ m/s. Using this value for c and taking the (measured) rest mass of an electron to be $9.109\,389 \times 10^{-31}$ kg, we find the rest energy of an electron to be 0.511 MeV, in agreement with the accepted value.

PROBLEM 2 – SOLUTIONS CONT.

The total energy is given by the relationship

$$E_{total} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

But $mc^2 = 0.511$ MeV, therefore

$$E_{total} = \frac{0.511\ \text{MeV}}{\sqrt{1 - \frac{(0.860c)^2}{c^2}}}$$

$$E_{total} = 1.00\ \text{MeV}$$

The total energy of the electron is 1.00 MeV.

$$E_{total} = E_K + E_{rest}$$

$$E_K = E_{total} - E_{rest}$$

$$= 1.00\ \text{MeV} - 0.511\text{MeV}$$

$$E_K = 0.489\ \text{MeV}$$

The kinetic energy of the electron is 0.489 MeV.

THE LIFE AND TIMES OF ALBERT EINSTEIN

- Born in the small town of Ulm, Germany
- Was not thought of as a bright child
- Did not qualify for graduate school or university postings after completing his undergraduate degree
- Worked for a clerk's office, where he published three papers
  - The photoelectric effect (in quantum theory)
  - The mathematical interpretation of Brownian motion (random motion of a particle in a fluid)
  - Special relativity
- Was subsequently offered university postings

THE LIFE AND TIMES OF ALBERT EINSTEIN – CONT.

- Deemed one of the “giants” of science alongside Sir Isaac Newton
- Was a visiting professor in California, but decided to stay permanently due to the rise of Hitler in Germany
- Instrumental in the development of the atomic bomb, but strongly opposed its use

SUMMARY – MASS-ENERGY: E=MC²

- Rest mass is the mass of an object at rest.
- Only the rest mass is used in relativistic calculations.
- Einstein's famous mass–energy equivalence equation is $E = mc^2$.
- The total energy of a particle is given by $E_{\text{total}} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$.
- Rest mass is a form of energy that is convertible into other more common and usable forms, for example, thermal energy.

PRACTICE

Readings

- Section 11.3 (pg 580)
- Section 11.4 (pg 585)

Questions

- pg 584 #2-5